



Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please
place your student identification label in this box

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work:
Working time:
minutes

ten minutes
one hundred

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	100	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (100 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(7 marks)

Sam is given the task of archiving 948 newspapers. On the first day he archived 5 newspapers, on the second day he archived 8 newspapers and on each subsequent day he archived 3 more newspapers than he did the day before.

- (a) Determine
- (i) how many newspapers Sam archived on the 5th day. (1 mark)

 - (ii) how many papers Sam had archived altogether after the 5th day. (1 mark)
- (b) On which day did Sam finish the task of archiving the newspapers and how many did he archive on that day? (2 mark)

After archiving the newspapers, Sam is given the task of refilling 50 toner cartridges. On the first day he refills 4 cartridges, leaving 46 remaining to be refilled, and continues to refill 4 cartridges on every subsequent day.

- (c) (i) Deduce a simplified rule for the number of cartridges remaining to be filled at the end of the n^{th} day. (2 marks)
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- (ii) Hence, or otherwise, determine the number of cartridges that remain to be filled at the end of the 6th day. (1 mark)

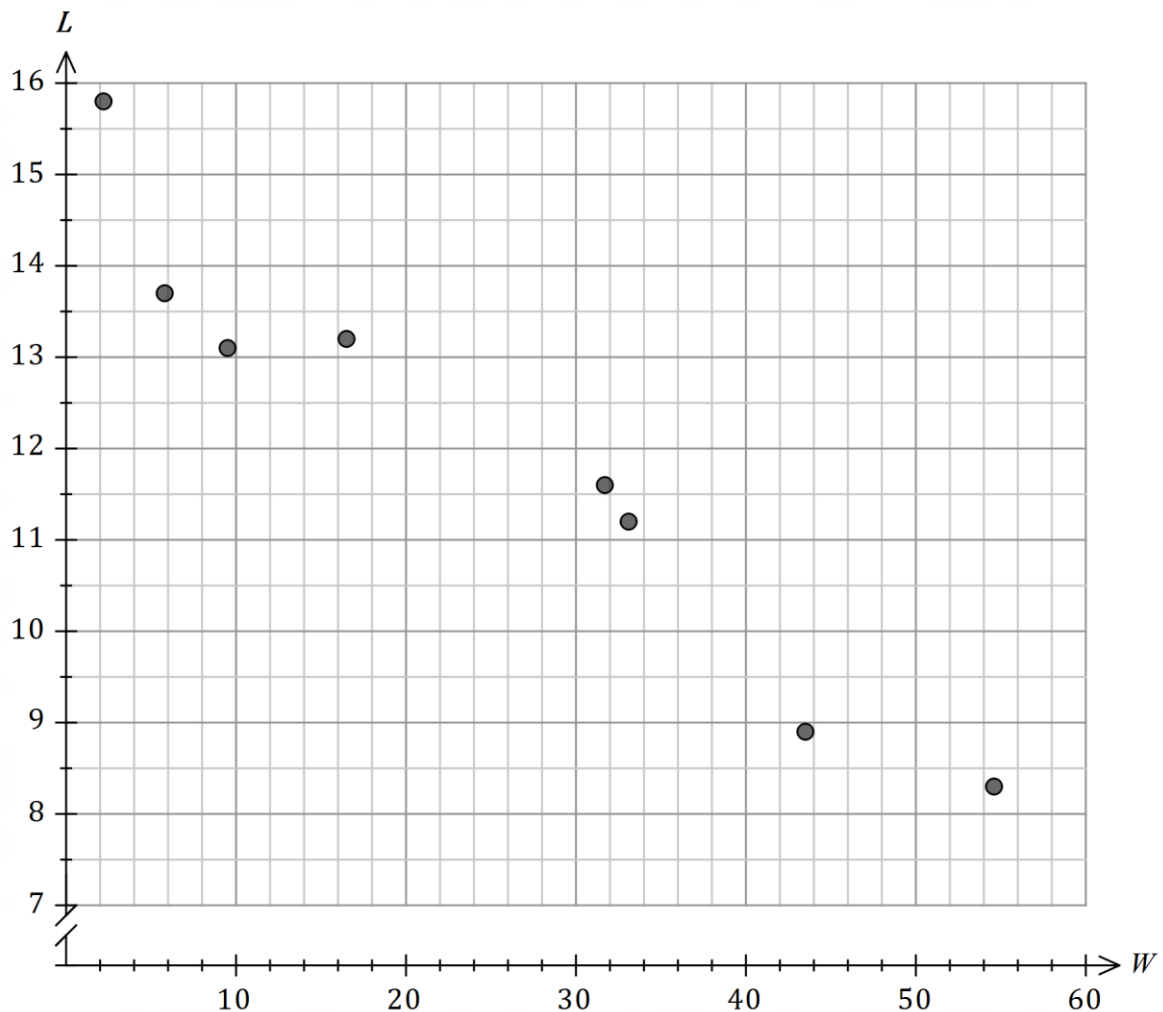
Question 10

(10 marks)

The table below shows the average lifespan L years and the average adult weight W kg of male dogs for a variety of breeds.

Breed	Weight W (kg)	Lifespan L (years)
English Setter	33.1	11.2
Jack Russell	5.8	13.7
Saint Bernard	43.5	8.9
Chihuahua	2.2	15.8
Beagle	9.5	13.1
Bullmastiff	54.6	8.3
Golden Retriever	31.7	11.6
Border Collie	16.5	13.2
German Shepherd	34.3	10.5
Spaniel	19.4	12.4

- (a) Complete the scatterplot below by plotting the data for German Shepherd's and Spaniel's. Clearly indicate these points on your graph. (1 mark)



See next page

- (b) Determine
- (i) the correlation coefficient between W and L . (1 mark)

 - (ii) the equation of the least-squares line that can be used to predict L from W . (2 marks)
- (c) Add the least-squares line to the scatterplot, clearly indicating the points used. (3 marks)
- (d) A breed of dog has an average male weight of 52.5 kg.
- (i) Predict the average lifespan of males of this breed. (1 mark)

 - (ii) Briefly discuss two factors that support the validity of your prediction. (2 marks)

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Question 11

(7 marks)

Six students were asked to create a short presentation to explain the meaning of some graph theory terms. The following table shows which terms each student offered to present.

Student	Terms
Fred	Circuit, Walk
Grace	Loop, Trail
Hazel	Loop, Trail, Edge
Joe	Trail
Kavan	Walk, Vertex
Mia	Edge, Vertex

- (a) Draw a clearly bipartite graph to represent this information. (3 marks)
- (b) Determine how many more edges must be added to the bipartite graph in (a) so that it would be a complete bipartite graph. (2 marks)
- (c) Draw another bipartite graph that is a subgraph of your graph in (a) to show how it is possible to assign each student to present just one term, so that all six terms are explained. (2 marks)

Question 12

(8 marks)

A random sample of 42 drivers was taken at a test centre. Each pair of letters shown below represents one driver. The first letter shows the driving test outcome (Pass, P or Fail, F) for the person and the second letter shows if they were taking the test for the first time (Yes, Y or N, No).

FN PN FN FY PY PN PY FY FN PY PN PY PY PN FY PY PN PN FN PN PN
FN FN PY FY PY FN FN FN FY PY FN FN PY FN PN PY PY **PN FY FY FN**

- (a) Two categorical variables have been recorded for each driver. Name one of the variables and explain why it is categorical. (2 marks)

- (b) The two-way frequency table below summarises the data. Complete the table by adding the last four results (bolded above) to the table. (2 marks)

	Yes	No
Pass		
Fail		

- (c) Convert the two-way frequency table to show column percentages. (2 marks)

%	Yes	No
Pass		
Fail		

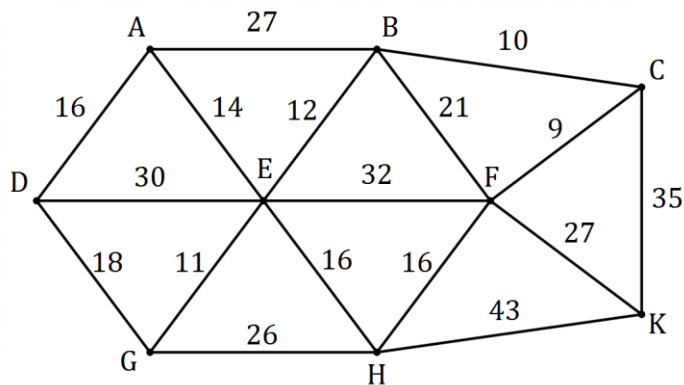
- (d) Discuss whether this sample data suggests the presence of an association between passing the driving test and taking the test for the first time. (2 marks)

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Question 13

(7 marks)

In the graph below, the vertices represent towns and the weights on each edge represent the distance, in kilometres, between pairs of towns. A parcel delivery service is based at town D.



- (a) Complete the table below to show the shortest distance d km from town D to each of the other towns. (4 marks)

Town	A	B	C	E	F	G	H	K
d , km								

- (b) State the route that gives the minimum distance between towns D and K. (1 mark)

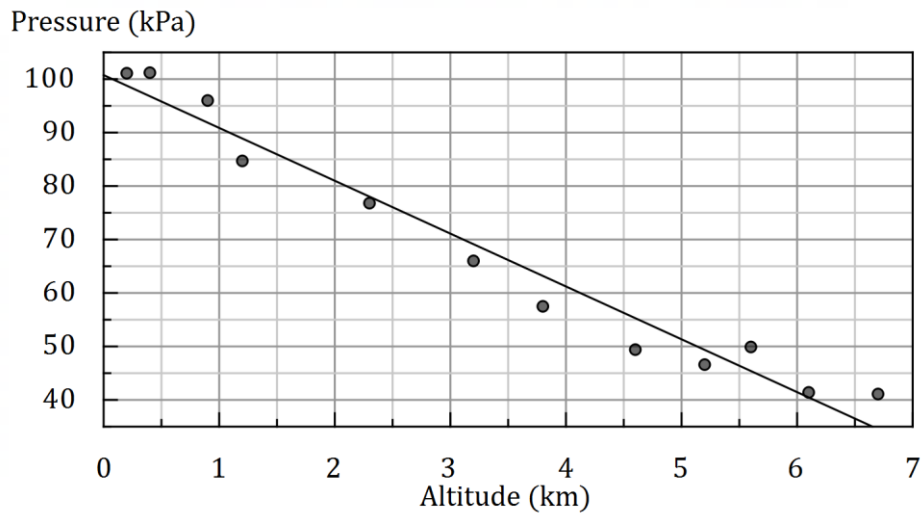
- (c) One day the delivery service has two parcels to deliver, one at A and the other at K. Determine the shortest path from D to K that passes through A and state the length of this path. (2 marks)

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Question 14

(7 marks)

The graph below shows pressure and altitude readings collected from a variety of sites in a country, together with the least-squares line for the linear association between the variables.



- (a) The correlation coefficient r for the linear association is one of the values shown in the list below. Circle this value and justify your choice. (3 marks)

{ -0.98, -0.82, -0.35, 0.03, 0.35, 0.82, 0.98 }

- (b) Determine the coefficient of determination for the linear association and interpret its value. (2 marks)

- (c) State, with reasons, whether the nature of the relationship between the variables is linear or non-linear. (2 marks)

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Question 15

(8 marks)

A photocopier was purchased for \$3 250. Its value depreciates at a rate of 4.5 cents per copy. The following recurrence relation can be used to determine V_n , the value of the photocopier in dollars after n copies have been made.

$$V_n = 3250 + b(n)$$

- (a) Explain why the value of b is -0.045. (2 marks)
- (b) Determine V_{400} . (1 mark)
- (c) Determine n when $V_n = 2647$. (2 marks)
- (d) Calculate the value of the photocopier after 44 000 copies have been made. (1 mark)
- (e) The photocopier will be replaced as soon as its value falls below \$350. Determine the number of copies the photocopier will make before it is replaced. (2 marks)

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Question 16

(6 marks)

The scores of a sample of students who sat two tests are shown in the table below.

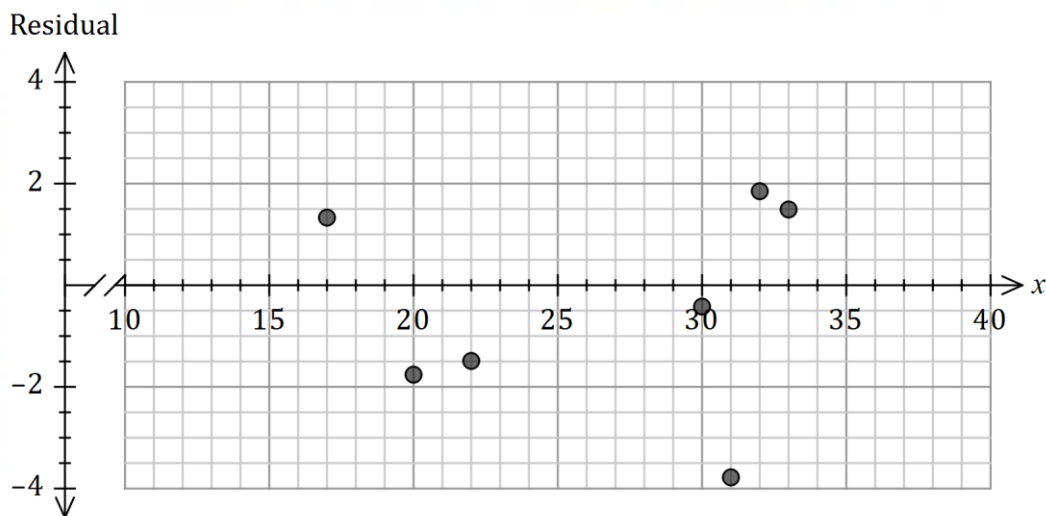
Student	1	2	3	4	5	6	7	8
Test A (x)	33	22	30	24	17	20	32	31
Test B (y)	14	18	14	21	24	19	15	10
Residual	1.49	-1.49	-0.42	p	1.33	-1.76	1.85	-3.78

Two students missed Test B and their teacher planned to predict their marks for this test using their scores from Test A and the linear relationship modelled by the least-squares line between the response (y) and explanatory (x) variables.

The equation is $\hat{y} = -0.635x + 33.46$ and the correlation coefficient is -0.867 . This equation was used to determine the residuals shown in the table above.

- (a) Determine the value of p in the table above. (2 marks)

- (b) Complete the residual plot by adding your residual from (a) to the plot below. (1 mark)



- (c) Using the residual plot and other relevant factors, comment on the teacher's plan. (3 marks)

Question 17

(10 marks)

Forrest John national park is home to a population of koalas whose numbers are increasing at a rate of 7.5% per year. In 2005 park rangers began a long-term study to monitor the changing number of koalas in the national park. They determined the initial population of koalas in 2005 to be 1600.

(a) State a recurrence relation for the koala population n years after 2005. (2 marks)

(b) (i) Determine the koala population in 2010. (1 mark)

(ii) Determine, to the nearest year, how long it will take for the koala population to approximately double. (2 marks)

(c) What assumption has been made in the calculations in parts (b)? (1 mark)

A bush fire at the end of 2010 in Forrest John national park reduces the amount of food available to the koalas. It is estimated that the population will begin to decrease at a rate of 17% per annum.

(d) Using your 2010 population from part (b)(ii) estimate the population of koalas in the park in 2018. (2 marks)

Starting in 2019, to combat the decline of koalas, park rangers decided to introduce 255 koalas to the park each year.

(e) What is the expected long-term population of koalas? (2 mark)

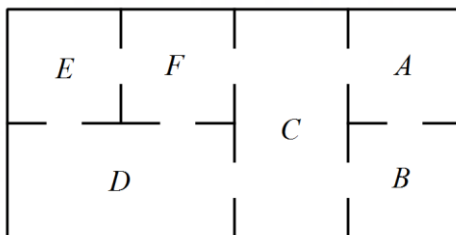
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Question 18

(8 marks)

A warehouse has dividing walls that split its interior into six areas, as shown in the plan below. The gaps in the dividing walls are doorways that allow people to move from one area to another.



- (a) Construct a graph to represent the warehouse areas and doorways, with each area being a vertex and each doorway an edge. (2 marks)
- (b) An inspector started in one area and followed a route that went through all doorways exactly once before stopping in another area. State where their route started and stopped and explain how the Eulerian properties of the graph in (a) help to identify these locations. (3 marks)
- (c) Another inspector wishes to start in an area, follow a route that visits all the other areas exactly once and end up back where they started. Comment on whether this is possible, referring to the Hamiltonian properties of the graph in (a) to justify your response. (3 marks)

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Question 19

(7 marks)

The average mid-year commuting times for full-time workers in Perth (p minutes) and Sydney (s minutes) between the years 2004 ($t = 4$) and 2011 ($t = 11$) are shown in the table below.

Year, t	4	5	6	7	8	9	10	11
Perth, p	24.2	25.8	27.2	26.1	27.7	27.1	30.2	29.5
Sydney, s	34.2	34.7	35.1	35.8	35.6	36.2	36.7	36.5

The least-squares line to model the linear relationship between t and s is $s = 0.348t + 33.0$ and $r_{ts} = 0.966$.

- (a) Determine the least-squares line to model the linear relationship between t and p and state the correlation coefficient for this association. (2 marks)
- (b) Predict the average commuting times in Perth and Sydney in the year 2021 and state, with justification, which prediction you are most confident in. (3 marks)
- (c) Predict the year in which the average commuting time will be the same in both cities and comment on how confident you are of this prediction. (2 marks)

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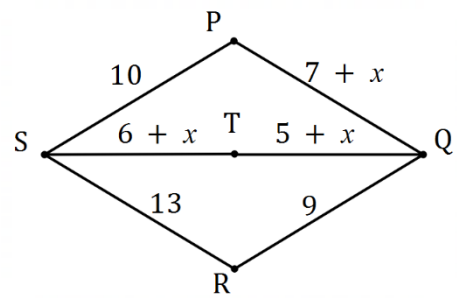
Question 20

(7 marks)

The edges in the graph (not to scale) represent roads and the weight on each edge is the time, in minutes, that it takes to drive along that road. The times to drive along PQ , ST and TQ vary throughout the day.

The variable x can only take whole number values.

An inspector wishes to drive along each road at least once, starting and finishing at P , in the minimum possible time.



- (a) Briefly explain why the edges on a path between S and Q will have to be repeated. (1 mark)
- (b) List all possible paths between S and Q , and state how long each would take, in terms of x where appropriate. (2 marks)
- (c) Determine all possible values of x so that ST would be one of the repeated edges. (2 marks)
- (d) For the case when $x = 7$, determine the time required for the inspectors' drive. (2 marks)

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Question 21

(8 marks)

A nail is hammered into a piece of wood. The distances moved by the tip of the nail during the first, second and third hits are 16, 12 and 9 mm respectively.

- (a) Show that the distances can be modelled by a geometric sequence. (2 marks)
- (b) Write a rule for the distance moved by the tip of the nail D_n during the n^{th} hit of the hammer in the form $D_n = a(r)^{n-1}$. (1 mark)
- (c) Determine which hit first moves the tip of the nail less than 3 mm, and state the distance moved during this hit, rounded to one decimal place. (2 marks)
- (d) The piece of wood is 60 mm thick. State, with justification, whether the tip of the nail will pass all the way through the piece of timber, stating any assumptions that you make. (3 marks)

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End of questions

Supplementary page

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